New Quantum Theory of Laser Cooling

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In this paper, we study the laser cooling mechanisms with new Schrodinger quantum wave equation, which can describe a particle in conservative and non-conservative force field. We prove the atom in laser field can be cooled with the new theory, and predict that the atom cooling temperature T is directly proportional to the atom vibration frequency ω, which is in accordance with experiment result.

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I. INTRODUCTION

During the last decade, significant progress has been achieved in laser cooling of lanthanides. Laser-cooled lanthanides are effectively used in such fundamental fields as the study of cold collisions [1], Bose-Einstein condensation [2], ultra-precise atomic clocks [3] and also open new perspectives for implementation in nano-technology [4] and quantum information [5]. In contrast to recently demonstrated method of buffer gas cooling and trapping of lanthanides in a magnetic dipole trap [6], high precision spectroscopy [7], atomic frequency standards [8–10], Bose-Einstein condensation [11], atomic nanolitography [12, 13] and so on.

Today, laser cooling of atoms is widely used in experiments where high precision spectroscopy or precise control of the atomic motion is necessary. A large variety of schemes have been proposed and applied to suit atoms with specific level structures and for obtaining particular temperature ranges [14]. In general, the lower the wanted temperature, the more sensitive does the light scattering process has to be on the velocity of the atom. For the most simple laser cooling scheme relying on the Doppler shift of an optical transition, Doppler laser cooling, this means the narrower the line width of the optical transition, the lower the obtainable temperature. However, since the maximum cooling force in the Doppler cooling scheme is dependent on the photon scattering rate, and narrow line width transitions will lead to longer cooling time than wider transitions.

In the theory of Laser cooling, there are semiclassical method for Sisyphus cooling [15], and showed that this method gives excellent agreement with the fully quantum- mechanical method [16]. In the semiclassical method the external degrees of freedom, i.e., position and momentum, are treated as simultaneously well-defined classical variables. The internal degree of freedom, i.e., the magnetic substate, is treated fully quantum mechanically, allowing for arbitrary superpositions.

On the other hand, various nano-mechanical resonators have been investigated [17] extensively in recent years. To reveal the quantum effect in the nano-mechanical devices, various cooling schemes [18–23] were proposed to drive them to reach the standard quantum limit [24]. A famous one among them is the optical radiation-pressure cooling scheme [12] attributed to the sideband cooling [20–23], which was previously well-developed to cool the spatial motion of the trapped ions [25] or the neutral atoms [26].

In this paper, we study the laser cooling mechanisms with new Schrodinger quantum wave equation, which can describe the particle in conservative and non-conservative force field [27]. We prove the atom can be cooled in laser field, i,e, the atom velocity approach zero, and find the cooling temperature of atom T is directly proportional to the atom vibration frequency ω, which is in accordance with experiment result.

II. THE RADIATION FORCE OF ATOM IN LIGHT FIELD

A moving atom sees the light moves towards Doppler shifted closer to resonance, whereas the light is shifted away from resonance. Thus, the atom predominantly scatters photons from the forward direction and is slowed down. As the Doppler effect plays a central role, the process is normally referred to as Doppler cooling. Although the cooling process is quantum mechanical in nature, as represented by the discrete momentum steps, the atomic motion may be treated classically, if the atomic wave-packet is well-localised in position and momentum space. In this case, the time-averaged interaction can be separated into a mean cooling force, and a diffusive term which accounts for the stochastic nature of the spontaneous emission. For Doppler cooling, the cooling force is generally obtained by treating the two beams independently. The extension of Doppler cooling to three dimensions is obvious. By using six beams, forming three orthogonal standing waves, an atom will everywhere see a viscous force.

Ring-like spatial distributions (modes) of atoms orbiting around a core were firstly observed in a misaligned cesium MOT [28] and explained in terms of the conventional MOT forces acting on each individual atom plus

the assumption about influence of the collective interatomic forces acting between the trapped atoms [28,29]. After observation in sodium MOT, the variety of spatial structures of cooled atoms, a simple model of coordinate-dependent vortex forces was developed which allowed to explain all observed cooled atoms structures and the transitions between them in terms of forces acting on each individual atom [30]. Due to the misalignment, the radiative force acting on the atom along the y direction has an x dependence and vice versa. In other words, beside the velocity and field-intensity dependent terms in the force expression an extra azimuthal component do appears, which is referred to as the vortex force. It is clear from consideration of the forces in xy plane. For a Gaussian beam propagating exactly along the x-direction, the velocity-independent part of the radiative force has the form [31]

$$\vec{F} = -k\vec{v} - \kappa\vec{r}, \tag{1}$$

where $\vec{v}$ is atom velocity, $\vec{r}$ is atom position, k is damp coefficient of atom in light field, κ is elastic recovery coefficient. The first term $-k\vec{v}$ is used for laser cooling, which is a non-conservative force, and the second term $-\kappa\vec{r}$ is used for laser trapping, which is a conservative force corresponding potential energy $\frac{1}{2}\kappa r^2$.

III. A NEW QUANTUM THEORY OF LASER COOLING

We know that Schrodinger equation is only suitable for the particle in conservative force field. For the particle in non-conservative field, it is needed new quantum wave equation describe it. Recently, we have proposed a new quantum wave equation, which can describe the particle in conservative and non-conservative force field [27]. It is

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = (-\frac{\hbar^2}{2m}\nabla^2 + U(r) - i\hbar\frac{3k}{m})\Psi(\vec{r},t) \tag{2}$$

where $U(r)$ is potential energy, the term $-i\hbar\frac{3k}{m}$ corresponding non-conservative force $\vec{F} = -k\vec{v}$. The Eq. (1) is the radiative force of atom in light field, which include both conservative force and non-conservative force, and it can be described by the equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = (-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}\kappa r^2 - i\hbar\frac{3k}{m})\Psi(\vec{r},t) \tag{3}$$

By the method of separation of variable

$$\Psi(\vec{r},t) = \Psi(\vec{r})f(t), \tag{4}$$

the Eq. (2) becomes

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r}) + \frac{1}{2}\kappa r^2\Psi(\vec{r}) = (E + i\hbar\frac{3k}{m})\Psi(\vec{r}), \tag{5}$$

and

$$f(t) = ce^{-\frac{i}{\hbar}Et}. \tag{6}$$

The wave function $\Psi(\vec{r})$ and energy E can be written plural form. let

$$\Psi(\vec{r}) = R(\vec{r}) + iS(\vec{r}), \tag{7}$$

and

$$E = E_1 + iE_2, \tag{8}$$

substituting Eqs. (7) and (8) into (5), we have

$$\begin{aligned} &-\frac{\hbar^2}{2m}\nabla^2 R(\vec{r}) - i\frac{\hbar^2}{2m}\nabla^2 S(\vec{r}) + \frac{1}{2}\kappa r^2 R(\vec{r}) + i\frac{1}{2}\kappa r^2 S(\vec{r}) \\ = \;& E_1 R(\vec{r}) - E_2 S(\vec{r}) - \hbar\frac{3k}{m}S(\vec{r}) \\ &+i(E_1 S(\vec{r}) + E_2 R(\vec{r}) + \hbar\frac{3k}{m}R(\vec{r})). \end{aligned} \tag{9}$$

From Eq. (9), we can obtain

$$-\frac{\hbar^2}{2m}\nabla^2 R(\vec{r}) = (E_1 - \frac{1}{2}\kappa r^2)R(\vec{r}) - (E_2 + \hbar\frac{3k}{m})S(\vec{r}), \tag{10}$$

and

$$-\frac{\hbar^2}{2m}\nabla^2 S(\vec{r}) = (E_1 - \frac{1}{2}\kappa r^2)S(\vec{r}) + (E_2 + \hbar\frac{3k}{m})R(\vec{r}), \tag{11}$$

Eq. (10) and (11) are multiplied by $R(\vec{r})$ and $S(\vec{r})$ respectively, we have

$$\begin{aligned} &-\frac{\hbar^2}{2m}\nabla^2 R(\vec{r}) \cdot R(\vec{r}) \\ = \;& (E_1 - \frac{1}{2}\kappa r^2)R^2(\vec{r}) - (E_2 + \hbar\frac{3k}{m})S(\vec{r}) \cdot R(\vec{r}), \end{aligned} \tag{12}$$

and

$$\begin{aligned} &-\frac{\hbar^2}{2m}\nabla^2 S(\vec{r}) \cdot S(\vec{r}) \\ = \;& (E_1 - \frac{1}{2}\kappa r^2)S^2(\vec{r}) + (E_2 + \hbar\frac{3k}{m})R(\vec{r}) \cdot S(\vec{r}), \end{aligned} \tag{13}$$

the sum of Eq. (12) and (13) is

$$\begin{aligned} &-\frac{\hbar^2}{2m}\nabla^2 R(\vec{r}) \cdot R(\vec{r}) - \frac{\hbar^2}{2m}\nabla^2 S(\vec{r}) \cdot S(\vec{r}) \\ = \;& (E_1 - \frac{1}{2}\kappa r^2)R^2(\vec{r}) + (E_1 - \frac{1}{2}\kappa r^2)S^2(\vec{r}), \end{aligned} \tag{14}$$

Eq. (14) can be written as

$$-\frac{\hbar^2}{2m}\nabla^2 R(\vec{r}) \cdot R(\vec{r}) = (E_1 - \frac{1}{2}\kappa r^2)R^2(\vec{r}), \tag{15}$$

and

$$-\frac{\hbar^2}{2m}\nabla^2 S(\vec{r}) \cdot S(\vec{r}) = ((E_1 - \frac{1}{2}\kappa r^2)S^2(\vec{r}), \tag{16}$$

Eq. (10) and (11) are multiplied by $S(\vec{r})$ and $R(\vec{r})$ respectively, we have

$$\begin{aligned}&-\frac{\hbar^2}{2m}\nabla^2 R(\vec{r})\cdot S(\vec{r})\\ &= (E_1-\frac{1}{2}\kappa r^2)R(\vec{r})\cdot S(\vec{r})-(E_2+\hbar\frac{3k}{m})S^2(\vec{r}),\end{aligned}\tag{17}$$

and

$$\begin{aligned}&-\frac{\hbar^2}{2m}\nabla^2 S(\vec{r})\cdot R(\vec{r})\\ &= (E_1-\frac{1}{2}\kappa r^2)S(\vec{r})\cdot R(\vec{r})+(E_2+\hbar\frac{3k}{m})R^2(\vec{r}),\end{aligned}\tag{18}$$

the minus of Eq. (17) and (18) is

$$\begin{aligned}&-\frac{\hbar^2}{2m}\nabla^2 R(\vec{r})\cdot S(\vec{r})+\frac{\hbar^2}{2m}\nabla^2 S(\vec{r})\cdot R(\vec{r})\\ &= -(E_2+\hbar\frac{3k}{m})(S^2(\vec{r})+R^2(\vec{r})),\end{aligned}\tag{19}$$

and divided by $R(\vec{r})\cdot S(\vec{r})$ in Eq. (19), we have

$$\begin{aligned}&-\frac{\hbar^2}{2m}\frac{\nabla^2 R(\vec{r})}{R(\vec{r})}+\frac{\hbar^2}{2m}\frac{\nabla^2 S(\vec{r})}{S(\vec{r})}\\ &= -(E_2+\hbar\frac{3k}{m})\frac{S(\vec{r})}{R(\vec{r})}-(E_2+\hbar\frac{3k}{m})\frac{R(\vec{r})}{S(\vec{r})}.\end{aligned}\tag{20}$$

From Eq. (15) and (16), we can find the left side of Eq. (20) is zero, and Eq. (20) can be written as

$$(E_2+\hbar\frac{3k}{m})(S^2(\vec{r})+R^2(\vec{r}))=0,\tag{21}$$

to get

$$E_2=-\hbar\frac{3k}{m}.\tag{22}$$

In the following, we should solve Eqs. (15) and (16), they can be written as

$$(-\frac{\hbar^2}{2m}\nabla^2+\frac{1}{2}\kappa r^2)R(\vec{r})=E_1 R(\vec{r}),\tag{23}$$

and

$$(-\frac{\hbar^2}{2m}\nabla^2+\frac{1}{2}\kappa r^2)S(\vec{r})=E_1 S(\vec{r}),\tag{24}$$

they are energy eigenequation of three-dimensional harmonic oscillator. In rectangular coordinate system, The Eqs. (23) and (24) eigenfunctions and eigenvalues are

$$R(\vec{r})=S(\vec{r})=\Psi_{n_x}(x)\Psi_{n_y}(y)\Psi_{n_z}(z),\tag{25}$$

and

$$E_1=E_N=(N+\frac{3}{2})\hbar\omega,\qquad N=0,1,2,3,\cdots\tag{26}$$

where $\Psi_{n_x}(x)$, $\Psi_{n_y}(y)$ and $\Psi_{n_z}(z)$ are the wave functions of one-dimensional harmonic oscillator. The Eq. (5) eigenfunction and eigenvalue is

$$\begin{aligned}&\Psi_{n_xn_yn_z}(x,y,z)\\ &= \Psi_{n_x}(x)\Psi_{n_y}(y)\Psi_{n_z}(z)+i\Psi_{n_x}(x)\Psi_{n_y}(y)\Psi_{n_z}(z)\end{aligned}\tag{27}$$

and

$$E=E_1+iE_2=(N+\frac{3}{2})\hbar\omega-i\hbar\frac{3k}{m},\tag{28}$$

the Eq. (3) particular solution is

$$\begin{aligned}&\Psi_{n_xn_yn_z}(x,y,z,t)=\Psi_{n_xn_yn_z}(x,y,z)e^{-\frac{i}{\hbar}Et}\\ &= \Psi_{n_xn_yn_z}(x,y,z)e^{-\frac{i}{\hbar}E_1t}\cdot e^{-\frac{3k}{m}t}.\end{aligned}\tag{29}$$

A atom velocity operator $\hat{v}$ is

$$\hat{v}=\frac{\hat{p}}{m}=\frac{\hbar}{m}\frac{1}{i}\nabla,\tag{30}$$

at the state $\Psi_{n_xn_yn_z}(x,y,z,t)$, the expectation value of velocity operator $\hat{v}$ is

$$\hat{v}(t)=\int\Psi^*_{n_xn_yn_z}(x,y,z,t)\hat{v}\Psi_{n_xn_yn_z}(x,y,z,t)d\vec{r},\tag{31}$$

the expectation value of velocity component operator $\hat{v}_x$ is

$$\begin{aligned}\bar{v}_x &= e^{-\frac{6k}{m}t}\int\Psi^*_{n_xn_yn_z}(x,y,z,t)\frac{\hbar}{m}\frac{1}{i}\frac{\partial}{\partial x}\Psi_{n_xn_yn_z}(x,y,z,t)dxdydz\\ &= e^{-\frac{6k}{m}t}\frac{\hbar}{m}\frac{1}{i}\int(\Psi^*_{n_x}(x)\Psi^*_{n_y}(y)\Psi^*_{n_z}(z)-i\Psi^*_{n_x}(x)\Psi^*_{n_y}(y)\Psi^*_{n_z}(z)\frac{\partial}{\partial x}(\Psi_{n_x}(x)\Psi_{n_y}(y)\Psi_{n_z}(z)+i\Psi_{n_x}(x)\Psi_{n_y}(y)\Psi_{n_z}(z))dxdydz\\ &= e^{-\frac{6k}{m}t}\frac{\hbar}{m}\frac{1}{i}\int(1-i)\Psi^*_{n_x}(x)\Psi^*_{n_y}(y)\Psi^*_{n_z}(z)(1+i)\frac{\partial}{\partial x}\Psi_{n_x}(x)\Psi_{n_y}(y)\Psi_{n_z}(z)dxdydz\\ &= e^{-\frac{6k}{m}t}\frac{\hbar}{m}\frac{2\alpha}{i}\int\Psi^*_{n_x}(x)\Psi^*_{n_y}(y)\Psi^*_{n_z}(z)\cdot(\sqrt{\frac{n_x}{2}}\Psi_{n_x-1}(x)-\sqrt{\frac{n_x+1}{2}}\Psi_{n_x+1}(x))\Psi_{n_y}(y)\Psi_{n_z}(z)dxdydz\\ &= e^{-\frac{6k}{m}t}\frac{\hbar}{m}\frac{\alpha}{i}\int\Psi^*_{n_x}(\sqrt{\frac{n_x}{2}}\Psi_{n_x-1}(x)-\sqrt{\frac{n_x+1}{2}}\Psi_{n_x+1}(x))dx\int\Psi^*_{n_y}(y)\Psi_{n_y}(y)dy\int\Psi^*_{n_z}(z)\Psi_{n_z}(z)dz=0,\end{aligned}\tag{32}$$

with $\alpha=\sqrt{\frac{m\omega}{\hbar}}$, similarly, there are

$$\bar{v}_y=0, \qquad \bar{v}_z=0, \tag{33}$$

the expectation value of velocity square component operator $\hat{v}_x^2$ is

$$\begin{aligned}
\overline{v_x^2} &= -\int \Psi^*_{n_xn_yn_z}(x,y,z,t)\frac{\hbar^2}{m^2}\frac{\partial^2}{\partial x^2}\Psi_{n_xn_yn_z}(x,y,z,t)dxdydz \\
&= -e^{-\frac{6k}{m}t}\frac{\hbar^2}{m^2}\int(\Psi^*_{n_x}(x)\Psi^*_{n_y}(y)\Psi^*_{n_z}(z)-i\Psi^*_{n_x}(x)\Psi^*_{n_y}(y)\Psi^*_{n_z}(z)) \\
&\qquad \frac{\partial^2}{\partial x^2}(\Psi_{n_x}(x)\Psi_{n_y}(y)\Psi_{n_z}(z)+i\Psi_{n_x}(x)\Psi_{n_y}(y)\Psi_{n_z}(z))dxdydz \\
&= -e^{-\frac{6k}{m}t}\frac{\hbar^2}{m^2}\int(1-i)\Psi^*_{n_x}(x)\Psi^*_{n_y}(y)\Psi^*_{n_z}(z)\frac{\partial^2}{\partial x^2}(1+i)\Psi_{n_x}(x)\Psi_{n_y}(y)\Psi_{n_z}(z)dxdydz \\
&= -e^{-\frac{6k}{m}t}\frac{2\hbar^2}{m^2}\int \Psi^*_{n_x}(x)\Psi^*_{n_y}(y)\Psi^*_{n_z}(z)\frac{\alpha^2}{2}(\sqrt{n_x(n_x-1)}\Psi_{n_x-2}(x)-(2n_x+1)\Psi_{n_x}(x) \\
&\qquad +\sqrt{(n_x+1)(n_x+2)}\Psi_{n_x+2}(x))\Psi_{n_y}(y)\Psi_{n_z}(z)dxdydz \\
&= e^{-\frac{6k}{m}t}\alpha^2\frac{\hbar^2}{m^2}\int \Psi^*_{n_x}(x)\Psi^*_{n_y}(y)\Psi^*_{n_z}(z)(2n_x+1)\Psi_{n_x}(x)\Psi_{n_y}(y)\Psi_{n_z}(z)dxdydz \\
&= e^{-\frac{6k}{m}t}\alpha^2\frac{\hbar^2}{m^2}\int(2n_x+1)\Psi^*_{n_x}(x)\Psi_{n_x}(x)dx\int\Psi^*_{n_y}(y)\Psi_{n_y}(y)dy\int\Psi^*_{n_z}(z)\Psi_{n_z}(z)dz \\
&= e^{-\frac{6k}{m}t}\frac{\hbar^2}{m^2}\frac{m\omega}{\hbar}(2n_x+1)=e^{-\frac{6k}{m}t}\frac{\hbar\omega}{m}(2n_x+1),
\end{aligned} \tag{34}$$

similarly, there are

$$\overline{v_y^2}=e^{-\frac{6k}{m}t}\frac{\hbar\omega}{m}(2n_y+1), \tag{35}$$

and

$$\overline{v_z^2}=e^{-\frac{6k}{m}t}\frac{\hbar\omega}{m}(2n_z+1), \tag{36}$$

From Eqs. (34)-(36), we can find when time t increases $\overline{v_x^2}\to 0$, $\overline{v_y^2}\to 0$ and $\overline{v_z^2}\to 0$, i.e., as time increases the atom in the laser field (particular solution (29)) should be cooled. In the following, we prove the atom can be cooled in general solution, and the general solution is

$$\begin{aligned}
\Psi(x,y,z,t) &= \sum_{n_xn_yn_z}C_{n_xn_yn_z}\Psi_{n_xn_yn_z}(x,y,z,t) \\
&= \sum_{n_xn_yn_z}C_{n_xn_yn_z}(1+i)\Psi_{n_x}(x)\Psi_{n_y}(y)\Psi_{n_z}(z) \\
&\qquad e^{-\frac{i}{\hbar}E_1t}e^{-\frac{3k}{m}t},
\end{aligned} \tag{37}$$

and the complex conjugate of the general solution is

$$\Psi^*(x,y,z,t)=\sum_{n'_xn'_yn'_z}C^*_{n'_xn'_yn'_z}(1-i)\Psi^*_{n'_x}(x)\Psi^*_{n'_y}(y)\Psi^*_{n'_z}(z)e^{-\frac{i}{\hbar}E'_1t}e^{-\frac{3k}{m}t}, \tag{38}$$

where $C_{n_xn_yn_z}$ and $C^*_{n'_xn'_yn'_z}$ are superposition coefficients, and E_1 and E'_1 are energy levels, they are

$$E_1=(n_x+n_y+n_z+\frac{3}{2})\hbar\omega, \qquad n_x,n_y,n_z=0,1,2,3,\cdots \tag{39}$$

and

$$E'_1=(n'_x+n'_y+n'_z+\frac{3}{2})\hbar\omega, \qquad n'_x,n'_y,n'_z=0,1,2,3,\cdots \tag{40}$$

the expectation value of velocity component operator $\hat{v}_x$ is

$$\begin{aligned}
\bar{v}_x &= e^{-\frac{6k}{m}t}\int\sum_{n_x'n_y'n_z'}\sum_{n_xn_yn_z}C^*_{n_x'n_y'n_z'}C_{n_xn_yn_z}(1-i)(1+i)\Psi^*_{n_x'}(x)\Psi^*_{n_y'}(y)\Psi^*_{n_z'}(z)\\
&\qquad\frac{\hbar}{m}\frac{1}{i}\frac{\partial}{\partial x}\Psi_{n_x}(x)\Psi_{n_y}(y)\Psi_{n_z}(z)e^{-\frac{i}{\hbar}(E_1-E_1')t}dxdydz\\
&= \frac{2\hbar}{m}\frac{1}{i}e^{-\frac{6k}{m}t}\sum_{n_x'n_y'n_z'}\sum_{n_xn_yn_z}C^*_{n_x'n_y'n_z'}C_{n_xn_yn_z}\int\Psi^*_{n_x'}(x)\Psi^*_{n_y'}(y)\Psi^*_{n_z'}(z)\\
&\qquad\alpha(\sqrt{\frac{n_x}{2}}\Psi_{n_x-1}(x)-\sqrt{\frac{n_x+1}{2}}\Psi_{n_x+1}(x))\Psi_{n_y}(y)\Psi_{n_z}(z)e^{-\frac{i}{\hbar}(E_1-E_1')t}dxdydz\\
&= \frac{2\hbar}{m}\frac{\alpha}{i}e^{-\frac{6k}{m}t}\sum_{n_x'n_y'n_z'}\sum_{n_xn_yn_z}C^*_{n_x'n_y'n_z'}C_{n_xn_yn_z}\int\Psi^*_{n_x'}(x)(\sqrt{\frac{n_x}{2}}\Psi_{n_x-1}(x)-\sqrt{\frac{n_x+1}{2}}\Psi_{n_x+1}(x))dx\\
&\quad\int\Psi^*_{n_y'}(y)\Psi_{n_y}(y)dy\Psi^*_{n_z'}(z)\Psi_{n_z}(z)dze^{-\frac{i}{\hbar}(E_1-E_1')t}\\
&= \frac{2\hbar}{m}\frac{\alpha}{i}e^{-\frac{6k}{m}t}\sum_{n_x'n_y'n_z'}\sum_{n_xn_yn_z}C^*_{n_x'n_y'n_z'}C_{n_xn_yn_z}\\
&\quad(\sqrt{\frac{n_x}{2}}\delta_{n_x',n_x-1}-\sqrt{\frac{n_x+1}{2}}\delta_{n_x',n_x+1})\delta_{n_y',n_y}\delta_{n_z',n_z}e^{-\frac{i}{\hbar}(E_1-E_1')t}\\
&= \frac{2\hbar}{m}\frac{\alpha}{i}e^{-\frac{6k}{m}t}\sum_{n_xn_yn_z}(\sqrt{\frac{n_x}{2}}C_{n_xn_yn_z}C^*_{n_x-1n_yn_z}e^{-\frac{i}{\hbar}(n_x-n_x+1)\hbar\omega t}\\
&\quad-\sqrt{\frac{n_x+1}{2}}C_{n_xn_yn_z}C^*_{n_x+1n_yn_z}e^{-\frac{i}{\hbar}(n_x-n_x-1)\hbar\omega t})\\
&= \frac{2\hbar}{m}\frac{\alpha}{i}e^{-\frac{6k}{m}t}\sum_{n_xn_yn_z}(\sqrt{\frac{n_x}{2}}C_{n_xn_yn_z}C^*_{n_x-1n_yn_z}e^{-i\omega t}-\sqrt{\frac{n_x+1}{2}}C_{n_xn_yn_z}C^*_{n_x+1n_yn_z}e^{i\omega t}),
\end{aligned}\tag{41}$$

the expectation value of velocity square component operator $\hat{v}_x^2$ is

$$\begin{aligned}
\overline{v_x^2} &= \int\Psi^*(x,y,z,t)(-\frac{\hbar^2}{m^2}\frac{\partial^2}{\partial x^2})\Psi(x,y,z,t)dxdydz\\
&= (-\frac{\hbar^2}{m^2})e^{-\frac{6k}{m}t}\int\sum_{n_x'n_y'n_z'}\sum_{n_xn_yn_z}C^*_{n_x'n_y'n_z'}C_{n_xn_yn_z}\Psi^*_{n_x'}(x)\Psi^*_{n_y'}(y)\Psi^*_{n_z'}(z)\\
&\quad(1-i)(1+i)\frac{\partial^2}{\partial x^2}(\Psi_{n_x}(x)\Psi_{n_y}(y)\Psi_{n_z}(z))e^{-\frac{i}{\hbar}(E_1-E_1')t}dxdydz\\
&= -\frac{2\hbar^2}{m^2}e^{-\frac{6k}{m}t}\sum_{n_x'n_y'n_z'}\sum_{n_xn_yn_z}C^*_{n_x'n_y'n_z'}C_{n_xn_yn_z}\int\Psi^*_{n_x'}(x)\Psi^*_{n_y'}(y)\Psi^*_{n_z'}(z)\frac{\alpha^2}{2}[\sqrt{n_x(n_x-1)}\Psi_{n_x-2}(x)\\
&\quad-(2n_x+1)\Psi_{n_x}(x)+\sqrt{(n_x+1)(n_x+2)}\Psi_{n_x+2}(x)]\Psi_{n_y}(y)\Psi_{n_z}(z)dxdydz\\
&= -\frac{\hbar^2\alpha^2}{m^2}e^{-\frac{6k}{m}t}\sum_{n_x'n_y'n_z'}\sum_{n_xn_yn_z}C^*_{n_x'n_y'n_z'}C_{n_xn_yn_z}[\sqrt{n_x(n_x-1)}\delta_{n_x',n_x-2}-(2n_x+1)\delta_{n_x',n_x}\\
&\quad+\sqrt{(n_x+1)(n_x+2)}\delta_{n_x',n_x+2}]\delta_{n_y',n_y}\delta_{n_z',n_z}e^{-\frac{i}{\hbar}(E_1-E_1')t}\\
&= -\frac{\hbar^2\alpha^2}{m^2}e^{-\frac{6k}{m}t}\sum_{n_xn_yn_z}[\sqrt{n_x(n_x+1)}C_{n_xn_yn_z}C^*_{n_x-2n_yn_z}e^{-\frac{i}{\hbar}(n_x-n_x+2)\hbar\omega t}\\
&\quad-(2n_x+1)C_{n_xn_yn_z}C^*_{n_xn_yn_z}e^{-\frac{i}{\hbar}(n_x-n_x)\hbar\omega t}\\
&\quad+\sqrt{(n_x+1)(n_x+2)}C_{n_xn_yn_z}C^*_{n_x+2n_yn_z}e^{-\frac{i}{\hbar}(n_x-n_x-2)\hbar\omega t}]
\end{aligned}$$

$$= -\frac{\hbar\omega}{m}e^{-\frac{6k}{m}t}\sum_{n_xn_yn_z}[\sqrt{n_x(n_x+1)}C_{n_xn_yn_z}C^*_{n_x-2n_yn_z}e^{-2i\omega t}$$
$$-(2n_x+1)C_{n_xn_yn_z}C^*_{n_xn_yn_z}+\sqrt{(n_x+1)(n_x+2)}C_{n_xn_yn_z}C^*_{n_x+2n_yn_z}e^{2i\omega t}], \tag{42}$$

In Eqs. (41) and (42), the series

$$\sum_{n_xn_yn_z}(\sqrt{\frac{n_x}{2}}C_{n_xn_yn_z}C^*_{n_x-1n_yn_z}e^{-i\omega t}-\sqrt{\frac{n_x+1}{2}}C_{n_xn_yn_z}C^*_{n_x+1n_yn_z}e^{i\omega t}), \tag{43}$$

and

$$\sum_{n_xn_yn_z}(\sqrt{n_x(n_x+1)}C_{n_xn_yn_z}C^*_{n_x-2n_yn_z}e^{-2i\omega t}-(2n_x+1)C_{n_xn_yn_z}C^*_{n_xn_yn_z}$$
$$+\sqrt{(n_x+1)(n_x+2)}C_{n_xn_yn_z}C^*_{n_x+2n_yn_z}e^{2i\omega t}), \tag{44}$$

are convergent, when time t increases $\hat{v}_x \to 0$ and $\hat{v}_x^2 \to 0$ ($\hat{v}_y \to 0$, $\hat{v}_y^2 \to 0$ and $\hat{v}_z \to 0$, $\hat{v}_z^2 \to 0$), i.e., as time increases the atom in the laser field (general solution (37)) should be cooled.
For three-dimensional harmonic oscillator, the wave functions are degenerate, and the degeneracy is

$$f=\frac{1}{2}(N+1)(N+2), \qquad N=0,1,2,3\cdots \tag{45}$$

the quantum number N and corresponding wave equation Ψ_N are

$N=0, \psi_{000}$,
$N=1, \psi_{100}, \psi_{010}, \psi_{001}$,
$N=2, \psi_{110}, \psi_{101}, \psi_{011}, \psi_{200}, \psi_{002}$,
$N=3, \psi_{111}, \psi_{102}, \psi_{120}, \psi_{210}, \psi_{021}, \psi_{012}, \psi_{201}, \psi_{300}, \psi_{030}, \psi_{003}$,
$N=4, \psi_{112}, \psi_{121}, \psi_{211}, \psi_{301}, \psi_{031}, \psi_{103}, \psi_{130}, \psi_{301}, \psi_{013}, \psi_{004}, \psi_{040}, \psi_{400}, \psi_{202}, \psi_{022}, \psi_{220}$,
$\cdots$

the total wave function can be written as:

$$\begin{aligned}\psi(x,y,z,t) &= (1+i)[C_0\psi_0(x,y,z)e^{-\frac{i}{\hbar}E_0t}e^{-\frac{3k}{m}t}+C_1\psi_1(x,y,z)e^{-\frac{i}{\hbar}E_1t}e^{-\frac{3k}{m}t} \\ &\quad +C_2\psi_2(x,y,z)e^{-\frac{i}{\hbar}E_2t}e^{-\frac{3k}{m}t}+\cdots+C_N\psi_N(x,y,z)e^{-\frac{i}{\hbar}E_Nt}e^{-\frac{3k}{m}t}+\cdots] \\ &= (1+i)e^{-\frac{3k}{m}t}[C_{000}\psi_0(x)\psi_0(y)\psi_0(z)e^{-i\frac{3}{2}\omega t}+C_{100}\psi_1(x)\psi_0(y)\psi_0(z)e^{-i\frac{5}{2}\omega t} \\ &\quad +C_{010}\psi_0(x)\psi_1(y)\psi_0(z)e^{-i\frac{5}{2}\omega t}+C_{001}\psi_0(x)\psi_0(y)\psi_1(z)e^{-i\frac{5}{2}\omega t} \\ &\quad +C_{110}\psi_1(x)\psi_1(y)\psi_0(z)e^{-i\frac{7}{2}\omega t}+C_{101}\psi_1(x)\psi_0(y)\psi_1(z)e^{-i\frac{7}{2}\omega t}+\cdots].\end{aligned} \tag{46}$$

The real measurement value of $\hat{v}_x^2$ is its average value in a period. It is

$$\begin{aligned}\langle\overline{v_x^2}\rangle &= \frac{1}{T}\int_0^T\overline{v_x^2}dt \\ &= \frac{\hbar\omega}{m}e^{-\frac{6k}{m}t}\sum_{n_xn_yn_z}(2n_x+1)C_{n_xn_yn_z}C^*_{n_xn_yn_z},\end{aligned} \tag{47}$$

from Eq. (47), we have

$$\begin{aligned}\langle\overline{v_x^2}\rangle &= \frac{\hbar\omega}{m}e^{-\frac{6k}{m}t}[|C_{000}|^2+3|C_{100}|^2+|C_{010}|^2+|C_{001}|^2 \\ &\quad +3|C_{110}|^2+3|C_{101}|^2|+|C_{011}|^2+5|C_{200}|^2+|C_{002}|^2+|C_{020}|^2 \\ &\quad +3|C_{111}|^2+3|C_{102}|^2+3|C_{120}|^2+5|C_{210}|^2+|C_{021}|^2+|C_{012}|^2 \\ &\quad +5|C_{201}|^2+7|C_{300}|^2+|C_{030}|^2+|C_{003}|^2 \\ &\quad +3|C_{112}|^2+3|C_{121}|^2+5|C_{211}|^2+7|C_{301}|^2+|C_{031}|^2 \\ &\quad +3|C_{103}|^2+3|C_{130}|^2+7|C_{301}|^2+|C_{013}|^2+|C_{004}|^2 \\ &\quad +|C_{040}|^2+9|C_{400}|^2+5|C_{202}|^2+|C_{022}|^2+5|C_{220}|^2+\cdots] \\ &= \frac{\hbar\omega}{m}e^{-\frac{6k}{m}t}[|C_{000}|^2+5|C_{100}|^2+14|C_{110}|^2+30|C_{111}|^2+55|C_{112}|^2+\cdots].\end{aligned} \tag{48}$$

According to Boltzmann distribution law, when the atom is at heat balance state, the probability that atom is on the energy level E is directly proportional to e^{-E/k_BT}.

The probability of atom ground state is

$$Ce^{-\frac{E_0}{k_BT}}, \tag{49}$$

the probability of atom first excited state is

$$Ce^{-\frac{E_1}{k_BT}}, \tag{50}$$

the probability of atom N-th excited state is

$$Ce^{-\frac{E_N}{k_BT}}, \tag{51}$$

the total probability is equal to 1, i.e.,

$$Ce^{-\frac{E_0}{k_BT}} + Ce^{-\frac{E_1}{k_BT}} + \cdots Ce^{-\frac{E_N}{k_BT}} + \cdots = 1, \tag{52}$$

and

$$C = \frac{1 - e^{-\frac{\hbar\omega}{K_BT}}}{e^{-\frac{E_0}{K_BT}}}. \tag{53}$$

From Eq. (46), we can calculate the states probability. The ground state probability is

$$(1+i)^2|C_{000}|^2 e^{-\frac{6k}{m}t} = Ce^{-\frac{E_0}{K_BT}} = 1 - e^{-\frac{\hbar\omega}{K_BT}}, \tag{54}$$

and

$$|C_{000}|^2 e^{-\frac{6k}{m}t} = \frac{1 - e^{-\frac{\hbar\omega}{K_BT}}}{2}, \tag{55}$$

for the first excited state, there is

$$\begin{aligned}(1+i)^2|C_{100}|^2 e^{-\frac{6k}{m}t} &= (1+i)^2|C_{010}|^2 e^{-\frac{6k}{m}t} \\ &= (1+i)^2|C_{001}|^2 e^{-\frac{6k}{m}t} \\ &= \frac{1}{3}Ce^{-\frac{E_1}{K_BT}} \\ &= \frac{1}{3}e^{-\frac{\hbar\omega}{K_BT}}(1 - e^{-\frac{\hbar\omega}{K_BT}}), \end{aligned} \tag{56}$$

and

$$\begin{aligned}|C_{100}|^2 e^{-\frac{6k}{m}t} &= |C_{010}|^2 e^{-\frac{6k}{m}t} = |C_{001}|^2 e^{-\frac{6k}{m}t} \\ &= \frac{1}{6}e^{-\frac{\hbar\omega}{K_BT}}(1 - e^{-\frac{\hbar\omega}{K_BT}}), \end{aligned} \tag{57}$$

for the second excited state, there is

$$2|C_{110}|^2 e^{-\frac{6k}{m}t} = \frac{1}{6}Ce^{-\frac{E_2}{K_BT}}, \tag{58}$$

and

$$|C_{110}|^2 e^{-\frac{6k}{m}t} = \frac{1}{12}e^{-\frac{2\hbar\omega}{K_BT}}(1 - e^{-\frac{\hbar\omega}{K_BT}}), \tag{59}$$

for the third excited state, there is

$$2|C_{111}|^2 e^{-\frac{6k}{m}t} = \frac{1}{10}Ce^{-\frac{E_3}{K_BT}}, \tag{60}$$

and

$$|C_{111}|^2 e^{-\frac{6k}{m}t} = \frac{1}{20}e^{-\frac{3\hbar\omega}{K_BT}}(1 - e^{-\frac{\hbar\omega}{K_BT}}), \tag{61}$$

for the forth excited state, there is

$$2|C_{112}|^2 e^{-\frac{6k}{m}t} = \frac{1}{15}Ce^{-\frac{E_4}{K_BT}}, \tag{62}$$

and

$$|C_{112}|^2 e^{-\frac{6k}{m}t} = \frac{1}{30}e^{-\frac{4\hbar\omega}{K_BT}}(1 - e^{-\frac{\hbar\omega}{K_BT}}), \tag{63}$$

substituting Eqs. (55), (57), (59), (61) and (63) into (48), we have

$$\begin{aligned}\langle \overline{v_x^2} \rangle &= \frac{\hbar\omega}{m}[\frac{1 - e^{-\frac{\hbar\omega}{K_BT}}}{2} + 5 \cdot \frac{1}{6}e^{-\frac{\hbar\omega}{K_BT}}(1 - e^{-\frac{\hbar\omega}{K_BT}}) + 14 \cdot \frac{1}{12}e^{-\frac{2\hbar\omega}{K_BT}}(1 - e^{-\frac{\hbar\omega}{K_BT}}) \\ &\quad + 30 \cdot \frac{1}{20}e^{-\frac{3\hbar\omega}{K_BT}}(1 - e^{-\frac{\hbar\omega}{K_BT}}) + 55 \cdot \frac{1}{30}e^{-\frac{4\hbar\omega}{K_BT}}(1 - e^{-\frac{\hbar\omega}{K_BT}}) + \cdots] \\ &= \frac{\hbar\omega}{m}(1 - e^{-\frac{\hbar\omega}{K_BT}})[\frac{1}{2} + \frac{5}{6}e^{-\frac{\hbar\omega}{K_BT}} + \frac{7}{6}e^{-\frac{2\hbar\omega}{K_BT}} + \frac{9}{6}e^{-\frac{3\hbar\omega}{K_BT}} + \frac{11}{6}e^{-\frac{4\hbar\omega}{K_BT}} + \cdots], \end{aligned} \tag{64}$$

we define the series S

$$S = \frac{1}{6}(5e^{-x} + 7e^{-2x} + 9e^{-3x} + 11e^{-4x} + 13e^{-5x} + \cdots), \tag{65}$$

with $x = \frac{\hbar\omega}{K_BT}$, and the series S_0 is

$$S_0 = 5e^{-x} + 7e^{-2x} + 9e^{-3x} + 11e^{-4x} + 13e^{-5x} + \cdots, \tag{66}$$

and

$$S_0 e^{-x} = 5e^{-2x} + 7e^{-3x} + 9e^{-4x} + 11e^{-5x} + 13e^{-6x} + \cdots , \tag{67}$$

so

$$\begin{aligned} S_0 - S_0 e^{-x} &= 5e^{-x} + 2e^{-2x} + 2e^{-3x} + 2e^{-4x} + 2e^{-5x} + 2e^{-6x} + \cdots \\ &= 5e^{-x} + 2 \cdot \lim_{N\to\infty} \frac{e^{-2x} - e^{(N+2)x}}{1 - e^{-x}} \\ &= \frac{5e^{-x} - 3e^{-2x}}{1 - e^{-x}}, \end{aligned} \tag{68}$$

and

$$S_0 = \frac{5e^{-x} - 3e^{-2x}}{(1 - e^{-x})^2}, \tag{69}$$

and so

$$S = \frac{1}{6} S_0 = \frac{1}{6} \frac{5e^{-x} - 3e^{-2x}}{(1 - e^{-x})^2}, \tag{70}$$

substituting Eq. (70) into (64), we have

$$\langle \overline{v_x^2} \rangle = \frac{\hbar\omega}{m} (1 - e^{-\frac{\hbar\omega}{K_B T}}) (\frac{1}{2} + \frac{1}{6} \frac{5e^{-\frac{\hbar\omega}{K_B T}} - 3e^{-\frac{2\hbar\omega}{K_B T}}}{(1 - e^{-\frac{\hbar\omega}{K_B T}})^2}). \tag{71}$$

The energy equipartition principle is

$$\frac{1}{2} m \langle \overline{v_x^2} \rangle = \frac{1}{2} k_B T, \tag{72}$$

substituting Eq. (71) into (72), we have

$$\frac{1}{2} \hbar\omega (1 - e^{-\frac{\hbar\omega}{K_B T}}) (\frac{1}{2} + \frac{1}{6} \frac{5e^{-\frac{\hbar\omega}{k_B T}} - 3e^{-\frac{2\hbar\omega}{K_B T}}}{(1 - e^{-\frac{\hbar\omega}{k_B T}})^2}) = \frac{1}{2} k_B T, \tag{73}$$

i.e.,

$$(1 - e^{-\frac{\hbar\omega}{K_B T}}) (\frac{1}{2} + \frac{1}{6} \frac{5e^{-\frac{\hbar\omega}{k_B T}} - 3e^{-\frac{2\hbar\omega}{K_B T}}}{(1 - e^{-\frac{\hbar\omega}{k_B T}})^2}) = \frac{k_B T}{\hbar\omega}. \tag{74}$$

Eq. (74) is atom cooling temperature equation in laser field, we can obtain the atom cooling temperature from the equation.

IV. NUMERICAL RESULT

Next, we present our numerical calculation of atom cooling temperature. The Eq. (44) is transcendental equation, we can obtain the cooling temperature by the following two functions crossing point

$$y_1 = (1 - e^{-\frac{\hbar\omega}{K_B T}}) (\frac{1}{2} + \frac{1}{6} \frac{5e^{-\frac{\hbar\omega}{k_B T}} - 3e^{-\frac{2\hbar\omega}{K_B T}}}{(1 - e^{-\frac{\hbar\omega}{k_B T}})^2}), \tag{75}$$

$$y_2 = \frac{k_B T}{\hbar\omega}. \tag{76}$$

The main input parameters are: Plank constant $\hbar = 1.05 \times 10^{-34} Js$, Boltzmann constant $k_B = 1.38 \times 10^{-23} JK^{-1}$, in laser field, the atom vibration frequency ω is about several hundred kHz. It exhibits that the functions y_1, y_2 varies with the temperature T in FIG. 1 to FIG. 3 corresponding to the different vibration frequencies ω. In FIG. 1, we take the vibration frequency $\omega = 100kHz$, and give the relation curve between functions y_1 , y_2 and temperature T, and then obtain the atom cooling temperature $T = 0.433425\mu K$. In FIG. 2, we take the vibration frequency $\omega = 500kHz$, and give the relation curve between functions y_1 , y_2 and temper-

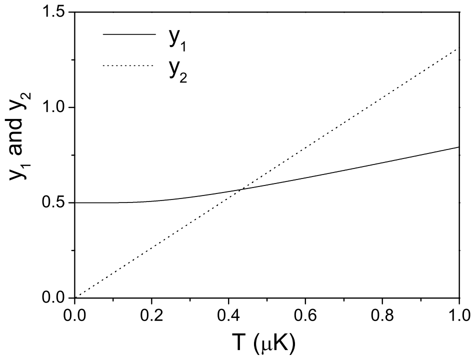

FIG. 1: The relation between y_1 , y_2 and temperature T when $\omega = 100kHz$.

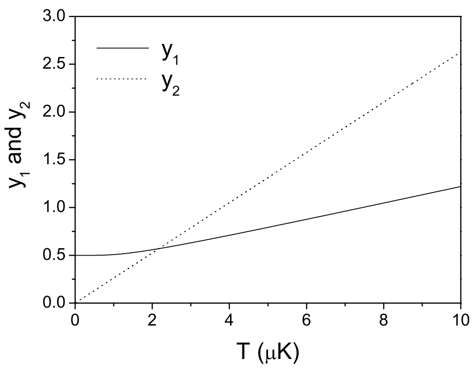

FIG. 2: The relation between y_1 , y_2 and temperature T when $\omega = 500kHz$.

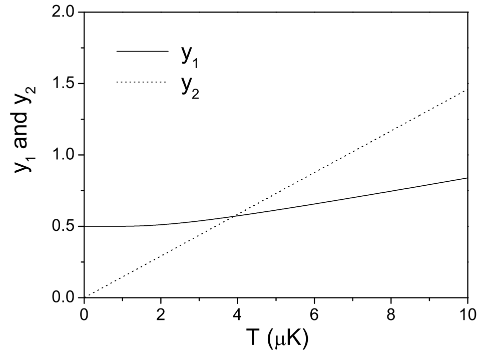

FIG. 3: The relation between y_1 , y_2 and temperature T when $\omega = 900kHz$.

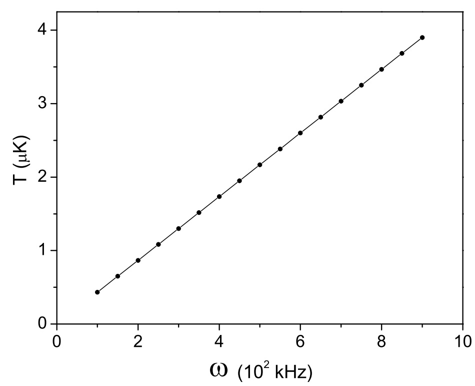

FIG. 4: The relation between temperature and frequency ω.

ature T, and then obtain the atom cooling temperature $T = 2.167125\mu K$. In FIG. 3, we take the vibration frequency $\omega = 900kHz$, and give the relation curve between functions y_1 , y_2 and temperature T, and then obtain the atom cooling temperature $T = 3.90082\mu K$. From FIG. 1 to FIG. 3, we can also find that the atom cooling temperature T gradually increase as the vibration frequency ω increase. In FIG. 4, we give the relation between vibration frequency ω and the atom cooling temperature T, and find the atom cooling temperature is directly proportional to vibration frequency. By calculation, we find the relation: $T = 4.334 \times 10^{-3}\omega$. In the formula, T unit is μK, and ω unit is KHz. When the atom vibration frequency ω is in the range of $100kHz \sim 900kHz$, the atom cooling temperature T is from $0.433\mu K$ to $3.901\mu K$. Recently, the authors A. D. O'Connell and so on measure atom cooling temperature $T = 25mK$ when the atom vibration frequency $\omega = 6.175GHz$. By the formula $T = 4.334 \times 10^{-3}\omega$, we obtain the atom cooling temperature $T = 26.76mK$. They have also find atom corresponding cooling temperature T is directly proportional to the atom vibration frequency ω [32]. These experiment results are in accordance with our theory results. Obviously, by reducing the atom vibration frequency ω, we can achieve more lower atom cooling temperature.

V. CONCLUSION

We study the laser cooling mechanisms with new Schrodinger quantum wave equation, which can describe the particle in conservative and non-conservative force field. With the new theory, We prove the atom can be cooled in laser field, and give the atom cooling temperature in laser field. Otherwise, we give new prediction: (1) the atom cooling temperature is directly proportional to the atom vibration frequency. By calculation, we find they are: $T = 4.334 \times 10^{-3}\omega$. (2) By reducing the atom vibration frequency in laser field, we can achieve more lower atom cooling temperature. these results are in accordance with recently experiment results.

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